Dr.SNS Rajalakshmi College of Arts and Science(Autonomous), Coimbatore-49

B.Sc. Mathematics CIA-I July'23

21UMA502 : Abstract Algebra

Registration Number : ________

DURATION : 3 Hours

MAXIMUM MARKS : 60

Section -A

(5x4=20)

1 .(a) Show that The inverse of each element of a group of a group is unique. (Or)

(b)Show that in a group G,$x^2 = xifandonlyifx = e$

2 .(a) Show that The identity element in a group is unique. (Or)

(b)Develop the properties of the group with illustration.

3 .(a) Define group with example. (Or)

(b)Show that The identity element in a group is unique.

4 .(a) Define Cyclic group (Or)

(b)State Lagrange's theorem

5.(a) Define congruence (Or)

(b)Define order of an element in a group.

Section -B

(5x6=30)

6 .(a) Show that G=\left\{\begin{pmatrix} 1_{} & 0_{} \\ 0_{} & 1_{} \end{pmatrix},\begin{pmatrix} -1_{} & 1_{} \\ -1_{} & 0_{} \end{pmatrix},\begin{pmatrix} 0_{} & -1_{} \\ 1_{} & -1_{} \end{pmatrix} \right\} is a group under matrix multiplication. (Or)

(b)Show that $G = \left\{a + b\sqrt{2}/a, b \in Z\right\}$ is a group under usual addition.

7 .(a) Let $G = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -1. \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right\}$. S.T G is a group under matrix multiplication. (Or)

(b)Let G be a Group. P.T (i) Identity element of G is unique. (ii) For any $a \in G$, the inverse of a is unique.

8 .(a) Let H be a subgroup of G. Then S.T a) the identity element of H is the same as of G.b)for each$a \in H$the inverse of a in H is same as the inverse of a in G. (Or)

(b)P.T A non empty subset H of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H.$

9 .(a) State and prove Euler's theorem (Or)

(b)If H & K are subgroups of a group G, then P.T $H \cap K$ is also a subgroup of G.

10 .(a) State and Prove Lagrange's theorem (Or)

(b)P.T Every subgroup of an abelian group is a normal subgroup.

Section -C

(Compulsory Question)

(1x10=10)

11. Let G be the set of all 2x2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $where a, b, c, d \in R, such that ad - bc = 1.$ P.T G is a group under matrix multiplication.